An Approach to “Just” Intonation By Employment of Difference Tones

by Albert Tipton

Why “Just” intonation? And, what are difference tones? To answer the first question, because it sounds better and sonority is greatly enhanced. With careful tuning, one can reach more sympathetic resonance and increase the carrying quality of the tone. Next, what are difference tones and what is the function of the ear in relation to them? In his book The Acoustical Foundations of Music (W.W. Norton, 1969, page 106), John Backus describes the phenomenon of difference tones as follows:

The first evidence that the ear is implicated came with the discovery, some two-hundred years ago, that when two fairly loud tones were sounded together, a third tone could be heard. For example, if loud tones of 300 cycles per second and 400 cycles per second are sounded simultaneously, the ear will hear a third tone of frequency 100 cycles per second. There is no tone of this frequency present in the original sound; a frequency meter will indicate only the presence of the original sounds of 300 and 400 cycles per second. Tones not actually present in the sound but heard by the ear are called SUBJECTIVE TONES. In our particular example, the frequency of the subjective tone is the difference of the two frequencies in the original sound; thus it is called a difference tone.

Because the difference tone is audible to the ear but does not register on a machine, we must train ourselves to hear it. In so doing, we notice that when the third sound is perfectly in phase, it reinforces the original sounds, enriching and even amplifying them. If imperfectly in phase, the third sound is out of focus, it disturbs the original sounds and ruins the inherent quality. Thus, a string quartet playing in perfect “just” intonation sounds rich, robust, and as though there were more than four instruments involved. By contrast, an out of tune or imperfectly tuned string quartet will sound thin, dry, and scratchy.

In this article, I suggest ways of achieving “just” intonation through self-testing, exercises, and a knowledge of the “CHORD OF NATURE,” the harmonic series. Determining and understanding the harmonic series is the foundation of “just” intonation. I am aware that accepted standards of pitch today range from A-430, as used in ensembles of period instruments, to A-442, and even A-444. The “just” intonation approach is applicable no matter which pitch standard is chosen. In fact, it is more important than ever where such flexibility exists.

Use any reliable tone generator such as a Korg, Seiko, or A-bell to certify the
standard of pitch. (I believe strongly in the supremacy of A-440, and this article adheres to that principle.) The normal range of flutes provides frequencies where it is easy to hear the difference tone phenomenon. Let us make certain that we tune accurately to A-440. Flutes are made to have a tuning leeway, to be pulled out a precise amount to play A-440 at 74 degrees Fahrenheit and at lower temperatures to be pushed in all the way if need be to bring up to pitch. The scale of the instrument is calibrated into 12 divisions within the octave, or tempered half steps.

As a function of its lengths determining the pitches, one may test the first important principle, that of playing in tune with the instrument. Assuming the instrument has been manufactured to play in tune with itself, first check the cork. Then determine how much the headjoint must be pulled out by comparing C² played with index finger of the left hand and played as the 2nd partial of the low C fingering (Example 1). (For me, with my instrument, the pulled-out position is ½".) These should be the same pitch. If not, either the instrument is out of tune with itself or the player is playing out of synchronization with the instrument. Now for an experiment, pull the head joint out excessively far and notice that the two C’s, as mentioned before, are noticeably different in pitch. This should be demonstration enough to plead the case of using a pitch standard to play in tune with instruments as they are originally made. To gain the best resonance of any instrument, a first requirement is that it be played in tune with itself.

Example 1

Next, let us get acquainted with the harmonic series or Chord of Nature (Example 2). Starting with the low C fingering, over blow to produce flute harmonics 1, 2, 3, 4, 5, 6, 7, 8 and piccolo harmonics 9, 10, 11, 12, 13, 14, 15, 16. Each higher harmonic is a multiple of the fundamental C = 261.6 x 2, x 3, x 4, etc. and is a smaller division of the vibrating air column. Therefore, the 2nd partial with the air column divided in half, beats twice as fast as the fundamental, 3rd partial, with the air column divided into 3rds beating three times as fast, etc. Each ascending interval is smaller in size.

Example 2

It is advisable to learn intervals by vibrating relationships. For instance, an octave is 1:2. In the tempered scale, in order to obtain 12 equidistant half steps within the octave we use the twelfth root of two, which is 1.05946. This system ignores the difference between whole steps (major and minor) occurring in “just” intonation and produces half steps smaller than is natural. A fifth is 2:3; a fourth is 3:4; major third 4:5; minor third is 5:6; smaller than a minor third but larger than a whole step is 6:7; 8:9 is our major whole step; 9:10 is our minor whole step; and 15:16 is the “just” half step.

To get back to our subject of difference tone tuning, notice that when two players play slightly out of tune, one hears beats; the farther apart the tuning disparity, the faster the beats. When the beats are fast enough, usually 30 cps, we
hear a third sound like a buzz. This is a difference tone which is the exact frequency differential between the two tones being played. Therefore, if any given fundamental X is used for the root of the Chord of Nature, that fundamental may be heard as the difference tone when contiguous intervals of the given chord are played simultaneously.

For instance, 3X with 4X = 1X(fundamental); 7X with 8X = 1X, etc. Like tuners and players, volume must be balanced and vibrato kept at a minimum in order to hear properly. When using two players, one player may make slight deviations in volume and pitch in order to perceive the difference tone. If one is able to achieve reinforcement of the relationships by playing in perfect tune, the consonant sonority is remarkably superior to the tempered tuning. It is possible to hear not only the fundamental but all the numbered harmonics as well.

In the Chord of Nature, notice the incidence of the fundamental's repetition at the octave 1:2 and 2:4 with 2:5 in between sounding the tonic three times and the dominant once before the 5th harmonic sounds the chordal major third in nature (see example 2). This coincides with the evolution of music from plain song to organum and much later to independent voice leading and the use of the major third.

The simplest approach to tonality is the use of the tonic and dominant steps of the scale to use in harmonization of the scale, avoiding 2nds and 7ths. Before attempting scales, however, let us see how well we can play in unison.

Let us play low C together. Now, one player holds low C while the other player attempts to play the numbered harmonic notes using normal fingering: C1, C2, C3, C4, C5. If the balance and tuning are correct, a greatly enhanced sound is obtained and a ghost of the series with notes played will be heard in the stationary low C of the other player. In most instances the low C will be flat, the 2nd C will be sharp, the 3rd harmonic C will be flat, the 4th C okay, but the 5th harmonic, E, will tend to be sharp. By raising the little finger of the right hand, the slight shading will effect a lower, or proper, tuning of the third. One player can practice with a tuner sounding the fundamental.

I advocate that two players play out of rhythmic synchronization to better identify each individual's pitch while a tuner is used to test the accuracy of both (Example 3). If the pitch is incorrect, do not fail to check the beginning pitch standard with an A-bell, tuner, or fork and avoid the impulse to change the tuning slide. When something isn't quite right in relationships, go back to the note departed from in order to gauge the contiguous relationships. The flute plays three and a half octaves: the 1st octave of fundamentals, the 2nd octave as 2nd partials, and the 3rd octave as combinations of 3rd, 4th, and 5th, even (in trills) 6th coincidental harmonics.

Example 3

Pitch alterations are made by turning in or out, covering more or less of the embouchure hole, by shading (skipping an adjacent hole and putting down one or more keys affects tuning on the flat side), or adding coincidental harmonics. For instance, a high E may be played as an overblown A to the 3rd series, a low E overblown to the 4th series, or a low C to the 5th series. By adding one or
more combinations of harmonics, the pitch is made sharper. Of course, alterations in volume affect pitch and the skilled player will do what is possible to disguise the problem.

In negotiating the pattern in Example 5, the 2nd C is not to be sharp or the low C-flat. This requires skill and a proper placement with the flute in the center of the tuning range. Turn in just far enough to get C² down but keep C¹ up. Then the G³ must be turned out with the left wrist. Having accomplished this perfectly, the next two notes, C and E, will be the same angle and properly in tune (Example 5).

Example 6 demonstrates the use of test tones with two players to be applied to any given tonality. Notice the resulting difference tones as indicated.
If in tune, then proceed with the exercises in Examples 7a, 7b and 8. These examples help master the following relationships:

2:3
4:5:6 triad
3:4:5, 2nd inversion
5:6, 1st inversion

For the artist, tuning is a constant concern that requires skillful adjustment at all times. Besides knowledge of the paths toward “just” intonation, one must know the idiosyncrasies of each instrument. Theobald Boehm redesigned the flute with a sensible method of key-coupling and application of theoretical physics. The changes he incorporated were modified by him empirically until he achieved the basic instrument most used today. No matter how perfectly each instrument’s scale is made, it takes an artist to play in tune.

In conclusion, one is readily able to hear the difference between “just” intonation without beats and the method of tempered tuning, which is a system of averaging out the error of 12 equal steps between octaves. Having decided on the value of fixed pitch instruments, which permit modulations to any key, one may ask, “why spend time on ‘just’ tuning?” I answer that, whenever possible, it is better, more pleasant, and enhancing to sonority when the discipline of “just” intonation is achieved. That means playing unisons and octaves in tune with the keyboard accompaniment and as well in tune as possible with any vertical configurations that occur.

Experience has, alas, proved that few players surpass the tuning error built into the tempered scale; however, when one hears a truly tuned a capella choir, a great string quartet, or a symphony orchestra playing in “just” intonation, the effect is truly marvelous.

ABOUT THE AUTHOR

ALBERT TIPTON (1917-1997) was born in Kansas City and raised in Tulsa, where his mother, Vena Tipton, was a professional flutist and teacher. He studied flute...
with William Kincaid at the Curtis Institute in Philadelphia, and was Assistant Solo Flutist of the Philadelphia Orchestra from 1940 to 1942.

Following a leave of absence for military service from 1943 to 1945, Tipton returned to the Philadelphia Orchestra. Subsequently, he became Solo Flutist of the St. Louis Symphony and of the Detroit Symphony. He was also an active conductor and formed his own chamber orchestra which toured from coast-to-coast for 20 years.

For over 40 years, Tipton brought distinction to the Aspen Music Festival as a flutist, conductor, and teacher. He also taught at Florida State University in Tallahassee and at the Shepherd School of Music at Rice University in Houston from which he retired in 1987 as Professor Emeritus.

At the 1995 convention of The National Flute Association in Orlando, Albert Tipton was honored by the NFA with the Lifetime Achievement Award for “total commitment to the integrity of his art—a commitment which has been manifest in his playing and his teaching throughout his career.”